

Hawking temperature for constant curvature black hole and its analogue in de Sitter Space

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Abstract

The constant curvature (CC) black holes are higher dimensional generalizations of BTZ black holes. It is known that these black holes have the unusual topology of $\mathcal{M}_{D-1} \times S^1$, where D is the spacetime dimension and \mathcal{M}_{D-1} stands for a conformal Minkowski spacetime in $D-1$ dimensions. The unusual topology and time-dependence for the exterior of these black holes cause some difficulties to derive their thermodynamic quantities. In this work, by using globally embedding approach, we obtain the Hawking temperature of the CC black holes. We find that the Hawking temperature takes the same form when using both the static and global coordinates. Also it is identical to the Gibbons-Hawking temperature of the boundary de Sitter spaces of these CC black holes. Employing the same approach, we obtain the Hawking temperature for the counterparts of CC black holes in de Sitter spaces.

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1 Introduction

Over the past years, the so-called BTZ (Banados-Teitelboim-Zanelli) [1] black hole solutions have played the important role in understanding microscopic degrees of freedom of black hole. The BTZ black hole is an exact solution of Einstein field equations with a negative cosmological constant in three dimensions. It is well known the BTZ black hole can be constructed by identifying points along the orbit of a Killing vector in a three dimensional anti-de Sitter (AdS) space.

The BTZ black hole has a topology of $\mathcal{M}_2 \times S^1$, where \mathcal{M}_2 denotes a conformal Minkowski space in two dimensions. Following the same way as done in three dimensions, one can construct analogues of the BTZ solution, the so-called constant curvature (CC) black holes in higher ($D \geq 4$) dimensional AdS spaces [2, 3, 4]. However, such black holes have topology of $\mathcal{M}_{D-1} \times S^1$ in D dimensions, which is quite different from the known topology of $\mathcal{M}_2 \times S^{D-2}$ for the usual black holes in D dimensions. In addition, the exterior region of these CC black holes is time-dependent and thus, there is no global time-like Killing vector [2]. Because of this, it is difficult to discuss Hawking radiation and thermodynamics associated with these black holes. For example, see [5, 6, 7] and references therein.

On the other hand, these spacetimes are interesting examples of smooth time-dependent solutions. Particularly, they are consistent background spacetimes for string theory at least to leading order since they are vacuum solutions to Einstein field equations with a negative cosmological constant too. Further we note that these spacetimes are time-dependent, the boundary metric is also time-dependent, and it is asymptotically AdS. Therefore, it might open a window to investigate dual strong coupling field theory in the time-dependent backgrounds through the AdS/CFT correspondence [8]. Especially, the D -dimensional CC black holes have the boundary topology of $dS_{D-2} \times S^1$, where dS_{D-2} denotes a $(D-2)$ -dimensional de Sitter (dS) space. Resorting the AdS/CFT correspondence, these CC black holes are gravity duals to strong coupling conformal field theories living on $dS_{D-2} \times S^1$. Finally it is observed in [9] that these CC black holes have a close connection to the so-called “bubbles of nothing” in AdS space [10, 11]. The bubbles of nothing were constructed by analytically continuing (Schwarzschild, Reissner-Nordström, and Kerr) black holes in AdS spaces. The stress-energy tensor for dual conformal field theories to these CC black holes was calculated in [9, 11].

It is well known that there is the Gibbons-Hawking temperature T_{GH} for a comoving observer in a dS space [12]. This temperature may be viewed as the Hawking temperature T_{HK} associated with cosmological horizon of dS space. A D -dimensional dS space can be

embedded as a hypersurface into a $(D + 1)$ -dimensional Minkowski space. Then, the comoving observer in dS space is identical to an observer with a constant acceleration in Minkowski space. According to Davies [13] and Unruh [14], an observer with a constant acceleration in Minkowski space will see a hot bath with the Davies-Unruh temperature $T_{DU} = a/2\pi$ where a is the acceleration of the observer. It turns out that the Gibbons-Hawking temperature of dS space is equivalent to the Davies-Unruh temperature of the corresponding observer in Minkowski space.

One decade ago, it was shown that an observer with a constant acceleration a in dS space will detect a temperature given by $\sqrt{a^2 + 1/l^2}/2\pi$, where l is the radius of the dS space [15]. This was soon generalized to the cases of dS/AdS space by Deser and Levin [16] with temperatures of $\sqrt{a^2 \pm 1/l^2}/2\pi$. Further, Deser and Levin have shown that the temperature is equivalent to the Davies-Unruh temperature for the corresponding observer in Minkowski space. Further examples for the equivalence have been shown by globally embedding curved spaces including BTZ, Schwarzschild, Schwarzschild-AdS (dS), and Reissner-Nordström solutions into higher dimensional Minkowski spaces in Ref.[17]. For more examples on the equivalence, see [18] and references therein.

In this work, the “globally embedding approach” will be employed to determine the Hawking temperature of CC black holes and positive CC spaces. This approach shows a clear way to compute the Hawking temperature, in comparison to other methods with ambiguity to calculate it.

In the next section, we show that the Hawking temperature of the constant curvature black holes is given by $T_{HK} = r_+/(2\pi l)$ using both the static and global coordinates in AdS spaces. Further it is shown that the Hawking temperature is identical to the Gibbons-Hawking temperature of the boundary dS space. In section 3 we consider the counterparts of the CC black holes in dS spaces. These are constant curvature (CC) spaces with the cosmological horizon. We find that the Hawking temperature for these spaces are given by $T_{HK} = r_+/(2\pi l)$, where r_+ and l are the cosmological horizon radius and the radius of dS spaces, respectively. We give our conclusions and discussions in section 4. In this paper, we confine ourselves to the five dimensional space. The generalization to other dimensions is straightforward.

2 Hawking temperature of CC black holes

A five dimensional AdS space is defined as the universal covering space of a surface obeying

$$-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 - x_5^2 = -l^2, \quad (2.1)$$

where l is the AdS radius. This surface has fifteen Killing vectors of seven rotations and eight boosts. We consider the boost $\xi = (r_+/l)(x_4\partial_5 + x_5\partial_4)$ with its norm $\xi^2 = r_+^2(-x_4^2 + x_5^2)/l^2$ where r_+ is an arbitrary real constant. The so-called CC black hole is constructed by identifying points along the orbit of the Killing vector ξ . Since the starting point is the AdS, the resulting black hole has a constant curvature as the AdS does show. The topology of the black holes is changed to be $\mathcal{M}_4 \times S^1$, which is quite different from the usual topology of $\mathcal{M}_2 \times S^3$ for five dimensional black holes. Here \mathcal{M}_n denotes a conformal Minkowski space in n dimensions. For more details for the construction of the black hole, see [3, 4].

The CC black holes can be nicely described by using Kruskal coordinates. For this purpose, a set of coordinates on the AdS for the region of $\xi^2 > 0$ has been introduced in Ref.[3]. The six dimensionless local coordinates (y_i, φ) are given by

$$\begin{aligned} x_i &= \frac{2ly_i}{1-y^2}, \quad i = 0, 1, 2, 3 \\ x_4 &= \frac{lr}{r_+} \sinh\left(\frac{r+\varphi}{l}\right), \\ x_5 &= \frac{lr}{r_+} \cosh\left(\frac{r+\varphi}{l}\right), \end{aligned} \quad (2.2)$$

with

$$r = r_+ \frac{1+y^2}{1-y^2}, \quad y^2 = -y_0^2 + y_1^2 + y_2^2 + y_3^2. \quad (2.3)$$

Here the allowed regions are $-\infty < y_i < \infty$ and $-\infty < \varphi < \infty$ with the restriction $-1 < y^2 < 1$. In these coordinates, the boundary at $r \rightarrow \infty$ corresponds to the hyperbolic “ball” which satisfies $y^2 = 1$. The induced line element can be written down

$$ds^2 = \frac{l^2(r+r_+)^2}{r_+^2}(-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + r^2 d\varphi^2. \quad (2.4)$$

Obviously, the Killing vector is given by $\xi = \partial_\varphi$ with its norm $\xi^2 = r^2$. The black hole spacetime could be obtained by identifying $\varphi \sim \varphi + 2\pi n$, and the topology of the black hole takes the form of $\mathcal{M}_4 \times S^1$ clearly.

On the other hand, the CC black holes can also be described by introducing Schwarzschild coordinates. The local “spherical” coordinates (t, r, θ, χ) in the hyperplane y_i are

$$\begin{aligned} y_0 &= f \sin \theta \sinh(r_+ t/l), & y_1 &= f \sin \theta \cosh(r_+ t/l), \\ y_2 &= f \cos \theta \sin \chi, & y_3 &= f \cos \theta \cos \chi, \end{aligned} \quad (2.5)$$

where $f = [(r - r_+)/r_+]^{1/2}$, $0 \leq \theta \leq \pi/2$, $0 \leq \chi \leq 2\pi$, and $r_+ \leq r < \infty$. One finds that the solution (2.4) can be rewritten as

$$ds^2 = l^2 N^2 d\Omega_3^2 + N^{-2} dr^2 + r^2 d\varphi^2, \quad (2.6)$$

where

$$N^2 = \frac{r^2 - r_+^2}{l^2}, \quad d\Omega_3^2 = -\sin^2 \theta dt^2 + \frac{l^2}{r_+^2} (d\theta^2 + \cos^2 \theta d\chi^2). \quad (2.7)$$

This is the black hole solution expressed in terms of Schwarzschild coordinates. Here $r = r_+$ is the location of black hole horizon. In these coordinates the solution seems static. However, we observe from (2.6) that the form (2.7) does not cover the whole exterior region of black hole since the difference of $y_1^2 - y_0^2$ is required to be positive in the region covered by these coordinates. Indeed, it has been proved that there is no globally timelike Killing vector in this geometry [2].

Now we consider a static observer with constant $(r > r_+, \theta, \chi, \varphi)$ in the black hole background (2.6). To this observer, we find that an associated acceleration a_5 is given by

$$a_5^2 = \frac{1}{l^2(r^2 - r_+^2)} \frac{1}{\sin^2 \theta} (r^2 \sin^2 \theta + r_+^2 \cos^2 \theta). \quad (2.8)$$

On the other hand, the acceleration of a_6 for the corresponding observer in six dimensional embedding Minkowski space is given by

$$a_6^{-2} = x_1^2 - x_0^2 = \frac{l^2(r^2 - r_+^2)}{r_+^2} \sin^2 \theta. \quad (2.9)$$

It is easy to check that these two accelerations obey the relation

$$a_6^2 = -\frac{1}{l^2} + a_5^2. \quad (2.10)$$

This shows that the Davies-Unruh temperature for the local observer in six dimensional Minkowski space is

$$T_{DU} = \frac{a_6}{2\pi} = \frac{r_+}{2\pi l \sqrt{r^2 - r_+^2}} \frac{1}{\sin \theta}. \quad (2.11)$$

We note that the redshift factor of $\sqrt{-g_{00}} = l N \sin \theta$ for the black hole (2.6) is necessary to define the Hawking temperature. Hence we conclude that the Hawking temperature of the CC black hole is

$$T_{HK} = \sqrt{-g_{00}} T_{DU} = \frac{r_+}{2\pi l}. \quad (2.12)$$

We notice that the Hawking temperature T_{HK} is consistent with the inverse period of the Euclidean time derived from the solution (2.7).

As the case in four dimensions [4], there is another set of coordinates covering the whole exterior of the Minkowskian black hole geometry as [9]

$$\begin{aligned} y_0 &= f \sinh(r_+ t/l), & y_1 &= f \cos \theta \cosh(r_+ t/l), \\ y_2 &= f \sin \theta \cos \chi \cosh(r_+ t/l), & y_3 &= f \sin \theta \sin \chi \cosh(r_+ t/l), \end{aligned} \quad (2.13)$$

where f is given by (2.5) and the allowed regions are $0 \leq \theta \leq \pi$, $r_+ \leq r < \infty$, and $0 \leq \chi \leq 2\pi$. In these coordinates, the solution can be expressed as

$$ds^2 = N^2 l^2 d\Omega_3^2 + N^{-2} dr^2 + r^2 d\varphi^2, \quad (2.14)$$

where $N^2 = (r^2 - r_+^2)/l^2$ and

$$d\Omega_3^2 = -dt^2 + \frac{l^2}{r_+^2} \cosh^2(r_+ t/l)(d\theta^2 + \sin^2 \theta d\chi^2). \quad (2.15)$$

The time-dependence of the solution is manifest in this coordinate system. We introduce a static observer located at constant $r > r_+$, φ and χ , but $\theta = 0$ due to the spherical symmetry of the solution [16]. Here, we find that the acceleration a_5 associated with the observer is

$$a_5^2 = \frac{r^2}{l^2(r^2 - r_+^2)}, \quad (2.16)$$

while the acceleration a_6 of the corresponding observer in six dimensional Minkowski space is given by

$$a_6^{-2} = x_1^2 - x_0^2 = \frac{r_+^2}{l^2(r^2 - r_+^2)}. \quad (2.17)$$

They satisfy the relation (2.10) too. In this case, the Davies-Unruh temperature is given by

$$T_{DU} = \frac{a_6}{2\pi} = \frac{r_+}{2\pi l \sqrt{r^2 - r_+^2}}. \quad (2.18)$$

Considering the redshift factor of $\sqrt{-g_{00}}$, we get the Hawking temperature of the black hole in the line element of (2.14) as

$$T_{HK} = \frac{r_+}{2\pi l}. \quad (2.19)$$

Thus we have obtained the Hawking temperature of the CC black hole by employing globally embedding approach combined with the Davies-Unruh temperature in six dimensional Minkowski space. The Hawking temperatures (2.12) and (2.19) are our main results.

Here some remarks are in order. First, in general, Hawking temperature of black hole depends on coordinates used to calculate it. That is, the Hawking temperature may be different when using different coordinates, even for the same black hole. In our case, we obtained the same Hawking temperature for the CC black hole even when used the different coordinate systems (2.6) and (2.14). Second, we mention that the Hawking temperature (2.12) is the same as the inverse period of the Euclidean time for

the Euclidean sector of the solution (2.6). However, when used the coordinates (2.15), the Hawking temperature is no longer the same as the inverse period of the Euclidean time. In order to see this, let us consider carefully the Euclidean sector of the black hole solution which can be obtained by replacing the time t by $-i(\tau + \pi l/(2r_+))$ in (2.15). In this case, $d\Omega_3^2$ becomes

$$d\Omega_3^2 = d\tau^2 + \frac{l^2}{r_+^2} \sin^2(r_+\tau/l)(d\theta^2 + \sin^2\theta d\chi^2). \quad (2.20)$$

In order that $d\Omega_3^2$ be a regular three-sphere, τ must have the period of $\tau \sim \tau + \tilde{\beta}$ with

$$\tilde{\beta} = \frac{\pi l}{r_+}. \quad (2.21)$$

Clearly this is not the inverse of Hawking temperature. This shows that the Euclidean method does not always provide a correct Hawking temperature of CC black holes. However, using both coordinates (2.5) and (2.13), the Euclidean time $\tau = it$ obtained by Wick rotation leads to the fact that it has a periodicity with period $2\pi l/r_+$, which gives a correct Hawking temperature of the black hole. This may be related to the issue of the factor 2 in [19].

Finally, we observe from (2.14) that the black hole solution has a boundary topology $dS_3 \times S^1$ at $r = \infty$. The three dimensional de Sitter space dS_3 has a Hubble constant $H = r_+/l$. It is well known that for a de Sitter space with a Hubble constant H , there is the Gibbons-Hawking temperature $T = H/2\pi$ for a comoving observer. We find that the Gibbons-Hawking temperature in our case is identical to the Hawking temperature of the CC black hole

$$T_{\text{GH}} = \frac{H}{2\pi} = \frac{r_+}{2\pi l} = T_{\text{HK}}. \quad (2.22)$$

3 Hawking temperature of a positive CC space

In this section we consider the analogue of the CC black hole in dS space. This space is constructed by identifying points along the orbit of a Killing vector in dS space. In fact, this space is a generalization of the three-dimensional Schwarzschild-de Sitter solution in higher dimensions. This space has a cosmological event horizon, and its topology is $\mathcal{M}_{D-1} \times S^1$ where \mathcal{M}_{D-1} denotes a $(D-1)$ -dimensional conformal Minkowski spacetime. Such space was constructed in Ref.[20].

As the case with a negative cosmological constant, we consider a five dimensional de Sitter space, which can be viewed as a hypersurface embedded into a six dimensional

Minkowski space, satisfying

$$-x_0^2 + x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = l^2 \quad (3.1)$$

with l the radius of the dS space. This dS space has fifteen Killing vectors of five boosts and ten rotations. We consider a rotational Killing vector $\xi = (r_+/l)(x_4\partial_5 - x_5\partial_4)$ with its norm $\xi^2 = r_+^2/l^2(x_4^2 + x_5^2)$ where r_+ is an arbitrary real constant. Identifying points along the orbit of a Killing vector ξ , another one-dimensional manifold becomes compact and it is isomorphic to S^1 . Thus, we obtain a spacetime of topology $\mathcal{M}^4 \times S^1$ with cosmological horizon. For details of the construction of the space, see [20].

We can describe the spacetime in the region with $0 \leq \xi^2 \leq r_+^2$ by introducing six dimensionless local coordinates (y_i, ϕ) ,

$$\begin{aligned} x_i &= \frac{2ly_i}{1+y^2}, \quad i = 0, 1, 2, 3 \\ x_4 &= \frac{lr}{r_+} \sin\left(\frac{r_+\phi}{l}\right), \\ x_5 &= \frac{lr}{r_+} \cos\left(\frac{r_+\phi}{l}\right), \end{aligned} \quad (3.2)$$

where

$$r = r_+ \frac{1-y^2}{1+y^2}, \quad y^2 = -y_0^2 + y_1^2 + y_2^2 + y_3^2. \quad (3.3)$$

Here the allowed regions are $-\infty < y_i < +\infty$ and $-\infty < \phi < +\infty$ with the restriction $-1 < y^2 < 1$ to have a positive r . In the coordinates (3.2), the induced line element is

$$ds^2 = \frac{l^2(r+r_+)^2}{r_+^2}(-dy_0^2 + dy_1^2 + dy_2^2 + dy_3^2) + r^2d\phi^2, \quad (3.4)$$

which is the same form as the case of a negative constant curvature [3]. However, it is noted that the coordinates (3.2) and the definition of r differ from those in the CC black holes. In this coordinate system, it is evident that the Killing vector is $\xi = \partial_\phi$ with norm $\xi^2 = r^2$. Imposing the identification $\phi \sim \phi + 2\pi n$, the solution has the topology $\mathcal{M}_4 \times S^1$.

We now introduce the Schwarzschild coordinates to describe the solution. Using local “spherical” coordinates (t, r, θ, χ) defined as

$$\begin{aligned} y_0 &= f \sin \theta \sinh(r_+ t/l), & y_1 &= f \sin \theta \cosh(r_+ t/l), \\ y_2 &= f \cos \theta \sin \chi, & y_3 &= f \cos \theta \cos \chi, \end{aligned} \quad (3.5)$$

where $f = [(r_+ - r)/(r + r_+)]^{1/2}$, and the allowed coordinate ranges are $0 < \theta < \pi/2$, $0 < \chi < 2\pi$, and $0 < r < r_+$. The line element can be expressed as

$$ds^2 = l^2 N^2 d\Omega_3^2 + N^{-2} dr^2 + r^2 d\phi^2. \quad (3.6)$$

Here $N^2 = (r_+^2 - r^2)/l^2$ and

$$d\Omega_3^2 = -\sin^2 \theta dt^2 + \frac{l^2}{r_+^2}(d\theta^2 + \cos^2 \theta d\chi^2). \quad (3.7)$$

Clearly the location of $r = r_+$ represents a cosmological horizon. This solution is the counterpart of a five dimensional CC black hole described in the previous section. The only difference is that $N^2 = (r^2 - r_+^2)/l^2$ is replaced by $N^2 = (r_+^2 - r^2)/l^2$ here. Further, in three dimensions, the corresponding induced line element takes the form

$$ds^2 = -(r_+^2 - r^2)dt^2 + \frac{l^2}{r_+^2 - r^2}dr^2 + r^2d\phi^2, \quad (3.8)$$

After a suitable rescaling of coordinates, it can be transformed to three dimensional Schwarzschild-de Sitter solution [21]. In this sense, the solution (3.6) can be viewed as an analogue of the three dimensional Schwarzschild-de Sitter solution in five dimensions.

The solution (3.6) seems to be static, but it does not cover the whole region within the cosmological horizon. It can be seen from the definition of coordinates (3.5) because they must obey the constraint: $y_1^2 - y_0^2 = f^2 \cos^2 \theta \geq 0$. Considering a static observer located at constant $(r < r_+, \theta, \chi)$ in the background (3.6), we find that the static observer has a constant acceleration a_5 as

$$a_5^2 = \frac{1}{l^2(r_+^2 - r^2)\sin^2 \theta} (r^2 \sin^2 \theta + r_+^2 \cos^2 \theta), \quad (3.9)$$

while the observer in six dimensional Minkowski space has a constant acceleration a_6 as

$$a_6^{-2} = x_1^2 - x_0^2 = \frac{l^2(r_+^2 - r^2)}{r_+^2} \sin^2 \theta. \quad (3.10)$$

These two accelerations are related to each other as

$$a_6^2 = 1/l^2 + a_5^2. \quad (3.11)$$

According to Davies and Unruh, the observer has a temperature as

$$T_{DU} = \frac{a_6}{2\pi} = \frac{r_+}{2\pi l \sqrt{r_+^2 - r^2} \sin \theta}. \quad (3.12)$$

Taking into account the redshift factor $\sqrt{-g_{00}}$ of the observer, one has the Hawking temperature as

$$T_{HK} = \frac{r_+}{2\pi l}. \quad (3.13)$$

On the other hand, one has another set of coordinates which covers the whole region within the cosmological horizon,

$$\begin{aligned} y_0 &= f \sinh(r_+ t/l), & y_1 &= f \cos \theta \cosh(r_+ t/l), \\ y_2 &= f \sin \theta \cos \chi \cosh(r_+ t/l), & y_3 &= f \sin \theta \sin \chi \cosh(r_+ t/l). \end{aligned} \quad (3.14)$$

In this case, the line element is described by

$$ds^2 = l^2 N^2 d\tilde{\Omega}_3^2 + N^{-2} dr^2 + r^2 d\phi^2, \quad (3.15)$$

where $N^2 = (r_+^2 - r^2)/l^2$ and

$$d\tilde{\Omega}_3^2 = -dt^2 + \frac{l^2}{r_+^2} \cosh^2(r_+ t/l) (d\theta^2 + \sin^2 \theta d\chi^2). \quad (3.16)$$

We consider a static observer located at constant position of $(r < r_+, \chi, \varphi)$ and $\theta = 0$. For such an observer, we have a constant acceleration a_5 as

$$a_5^2 = \frac{r^2}{l^2(r_+^2 - r^2)}. \quad (3.17)$$

In six dimensional Minkowski space, the acceleration a_6 associated with the corresponding observer takes the form

$$a_6^{-2} = x_1^2 - x_0^2 = \frac{l^2(r_+^2 - r^2)}{r_+^2}. \quad (3.18)$$

We check that they obey the relation $a_6^2 = a_5^2 + 1/l^2$. We conclude that the observer has the Davies-Unruh temperature

$$T_{DU} = \frac{a_6}{2\pi} = \frac{r_+}{2\pi l \sqrt{r_+^2 - r^2}}. \quad (3.19)$$

Considering the redshift factor for the observer in the background (3.15), we have the Hawking temperature observed as

$$T_{HK} = \frac{r_+}{2\pi l}. \quad (3.20)$$

Consequently, we find the same Hawking temperature as (3.13) obtained when using the coordinates (3.6).

4 Conclusions and Discussions

A D -dimensional CC black hole has unusual topological structure $\mathcal{M}_{D-1} \times S^1$ and there is no globally timelike Killing vector in the geometry of the black hole. Hence it was quite

difficult to discuss thermodynamic properties and Hawking temperature associated with this black hole.

For example, Banados has considered a five dimensional rotating CC black hole and embedded it into a Chern-Simons supergravity theory [3]. By computing related conserved charges, it was shown that the black hole mass is proportional to the product of outer horizon r_+ and inner horizon r_- , while the angular momentum is proportional to the sum of two horizons. In this case, the entropy of black hole is found to be proportional not to the outer horizon r_+ but the inner horizon r_- . This approach has two drawbacks. One is that the result cannot be degenerated to the non-rotating case. The other is that it cannot be generalized to other dimensions.

Creighton and Mann have considered the quasilocal thermodynamics of a four dimensional CC black hole in general relativity by computing thermodynamic quantities at a finite boundary which encloses the black hole [5]. They have shown that the entropy is not associated with the event horizon, but the Killing horizon of a static observer which is tangent to the event horizon of the black hole. The quasilocal energy density [see (11) of [5]] is negative.

In this work, we have derived Hawking temperature of CC black holes by employing the globally embedding approach since these black holes can be embedded into higher dimensional Minkowski space. We found that the Hawking temperature of CC black holes is given by $r_+/2\pi l$ when using both static and global coordinates. Here r_+ and l are black hole horizon and the radius of AdS space. Furthermore we found that the Hawking temperature is also identical to the Gibbons-Hawking temperature of the boundary dS space of the CC black holes. Importantly, we mention that the Hawking temperature obtained in this work is the same as that obtained from semi-classical tunneling method [19]. It turns out that the globally embedding technique is powerful to determine the Hawking temperature of CC black hole without any ambiguity. Using the same approach, we also obtained Hawking temperature of a positive CC space which is counterpart of CC black hole in dS space.

Finally, we comment that those solutions including CC black holes and positive CC spaces depend on an arbitrary real constant r_+ . The r_+ -dependence can be made disappear by rescaling coordinates. In this case, the Hawking temperature is given by $1/2\pi l$.

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